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Optimum Hypersonic Lifting Wings

FINAL REPORT[1]

by

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1. INTRODUCTION

The object of this investigation is a general analysis of optimum affine, two-dimensional, or three-dimensional lifting wings in hypersonic flow. Physically, Newtonian theory is employed. Mathematically, the methods of the theory of maxima and minima and those of the calculus of variations in one or two independent variables are used in order to maximize the lift-to-drag ratio for given geometric and aerodynamic constraints. This quantity is important in that the range and the maneuverability of a hypersonic cruise vehicle, a hypersonic glide vehicle, and a reentry vehicle increase linearly with it. Since flat-top wings are naturally suited to produce high lift-to-drag ratios at hypersonic speeds, particular attention is devoted to these wings under the assumptions that the free-stream velocity is parallel to the flat top, the pressure distribution is modified Newtonian, and the surface-averaged skin-friction coefficient is constant.

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2. AFFINE WINGS

These wings are such that the chordwise contour of any airfoil can be generated from the root contour by means of a linear transformation not involving rotation.

2.1. Direct Methods. Affine wings whose chordwise thickness distribution is a power law and whose spanwise thickness distribution is proportional to some power of the chord distribution are considered in Ref. 1. Then, the effect of the thickness ratio and the power law exponents on the lift-to-drag ratio is investigated. It is shown that a set of values of the thickness ratio and the power law exponents exists which yields a maximum lift-to-drag ratio. Specifically, the optimum thickness ratio is such that the friction drag is one-third of the total drag; the optimum chordwise power law exponent is one, meaning that a linear thickness distribution is the best in the chordwise sense; and the optimum spanwise power law exponent is one, meaning that a thickness distribution proportional to the chord distribution is the best in the spanwise sense. For a friction coefficient $C_f = 10^{-3}$, the maximum lift-to-drag ratio is $E = 5.29$ and corresponds to a thickness ratio $\tau = 0.126$.

2.2. Indirect Methods. The problem considered in Ref. 1 is investigated once more with reference to affine wings of arbitrary chordwise and spanwise contours (Ref. 2). By means of the indirect methods of the calculus of variations, it is proved that the solutions of Ref. 1 are variational solutions. A paper based on Ref. 2 has appeared in the Zeitschrift für Flugwissenschaften (Ref. 11).

2.3. Similarity Laws. In the previous investigations, the maximum lift-to-drag ratio was determined assuming that no constraints are imposed on the configuration. In a practical design, requirements may be imposed on the lift, the planform area, the frontal area, the volume, the root chord, the span, and the root thickness. Since the number of possible variational problems is practically without limit, economy of thought leads one to pose the following questions: (1) Is there any similarity law which permits one to determine the optimum chordwise contour of a wing of arbitrary spanwise contour from the known optimum chordwise contour of a reference wing? and (2) Is there any similarity law which permits one to determine the optimum spanwise contour of a wing of arbitrary chordwise contour from the known optimum spanwise contour of a reference wing? The answer to these questions can be found in Ref. 3 where two similarity laws are derived.

The Similarity Law for Chordwise Contours permits one to determine the optimum chordwise contour of a wing of arbitrary spanwise contour and chord distribution from the known optimum chordwise contour of a reference wing (a wing of constant trailing edge thickness and constant chord); the aerodynamic and geometric quantities of the latter must be replaced by appropriate proportional quantities of the former, with the proportionality constants depending only on the prescribed spanwise contour and chord distribution.

The Similarity Law for Spanwise Contours permits one to determine the optimum spanwise contour and chord distribution of a wing of arbitrary chordwise contour from the known optimum spanwise contour and chord distribution of a reference wing (a wing with a linear chordwise thickness distribution); the aerodynamic and geometric quantities of the latter must be replaced by appropriate proportional quantities of the former, with the proportionality constants depending only on the prescribed chordwise contour.

3. TWO-DIMENSIONAL WINGS

These wings are such that the chordwise contour of any airfoil is identical with the root contour.

3.1. Direct Methods. Two-dimensional wings whose chordwise contour is a power law are considered in Ref. 4, and the combination of power law exponent and thickness ratio maximizing the lift-to-drag ratio is determined. First, unconstrained configurations are considered, and it is shown that the optimum thickness ratio is such that the skin-friction drag is one-third of the total drag. For a surface-averaged skin-friction coefficient $C_f = 10^{-3}$, the maximum lift-to-drag ratio is $E = 5.29$ and corresponds to a wedge of thickness ratio $\tau = 0.126$. Next, constrained configurations are considered, that is, conditions are imposed on the length, the thickness, the enclosed area, and the position of the center of pressure. For each combination of constraints, an appropriate similarity parameter is introduced, and the optimum power law exponent, thickness ratio, and lift-to-drag ratio are determined as functions of the similarity parameter.

3.2. Indirect Methods. Two-dimensional wings of arbitrary chordwise contour are investigated in Ref. 5 and 6. First, it is assumed that the lift is a prescribed quantity (Ref. 5), and the necessary conditions to be satisfied by a minimum drag airfoil are derived for conditions imposed on the length, the thickness, the profile area, and the pitching moment. Then, the following particular cases are analyzed: (a) given lift, (b) given lift and chord length, (c) given lift and thickness, (d) given lift and profile area, and (e) given lift, pitching moment, and chord length. In all of these cases, analytical expressions are presented for the geometry of the optimum airfoil and the aerodynamic drag. Next, the lift is regarded to be unconstrained (Ref. 6), and the necessary conditions to be satisfied by a maximum lift-to-drag ratio airfoil are derived for conditions imposed on the length, the thickness, and the profile area. Once more, several particular cases are studied and, for each of these, analytical expressions are determined for the optimum shape and the maximum lift-to-drag ratio. A paper based on Ref. 6 has appeared in the Journal of the Astronautical Sciences (Ref. 12).

4. THREE-DIMENSIONAL WINGS

These wings are characterized by the fact that their chordwise and spanwise contours are arbitrary. The investigation of these wings requires the use of the indirect methods of the calculus of variations in two independent variables.

First, it is assumed that the lift is prescribed (Ref. 7), and the necessary conditions to be satisfied by a minimum drag wing are derived for (a) unconstrained volume and (b) given volume. For case (a), the optimum wing has a constant chordwise slope and a trailing edge thickness distribution similar to the chord distribution. While the planform area is uniquely determined, the chord distribution is not. In other words, there exist an infinite number of chord distributions yielding the same maximum value of the lift-to-drag ratio. For case (b), two solutions are possible depending on the value of the volume-lift parameter, a parameter directly proportional to the volume and inversely proportional to the lift squared. If the volume-lift parameter is greater than a certain critical value, the optimum wing is identical with that of case (a). If the volume-lift parameter is smaller than the critical value, the optimum wing has a constant chord and a constant trailing edge thickness. Also, the chordwise slope is constant in the spanwise sense but not in the chordwise sense. Finally, the maximum lift-to-drag ratio decreases as the volume-lift parameter decreases.

Next, the lift is regarded to be unconstrained (Refs. 8 and 9), and the necessary conditions to be satisfied by a maximum lift-to-drag ratio wing are derived for (a) unconstrained volume and (b) given volume. For case (a), the optimum wing surface is unique; it has a constant chordwise slope and a trailing edge thickness distribution similar to the chord distribution. For case (b), the chordwise slope of the optimum wing is constant in the spanwise sense but not in the chordwise sense. A one-parameter family of extremal solutions exists, depending on the value of the volume parameter: this parameter is directly proportional to the volume and inversely proportional to the span and the root chord squared. If the volume parameter exceeds a certain critical value, the optimum wing is convex. If the volume parameter is equal to the critical value, the optimum wing is identical with that of case (a). Finally, if the volume parameter is smaller than the critical value, the optimum wing is slightly concave. A paper based on Refs. 8 and 9 has appeared in the Journal of the Astronautical Sciences (Ref. 13).

5. BODIES

In order to complete the studies carried out under NASA Grant No. NGR-44-006-034, an investigation of the lift-to-drag ratio attainable by a slender, flat-top, homothetic body flying at hypersonic speeds is undertaken in Ref. 10. It is shown that a value of the thickness ratio exists such that the lift-to-drag ratio is a maximum; this particular value is such that the friction drag is one-third of the total drag. The subsequent optimization of the longitudinal and transversal contours is reduced to the extremization of products of powers of integrals related to the lift, the pressure drag, and the skin-friction drag. With regard to the longitudinal contour, a conical solution is the best. With regard to the transversal contour, the optimum solution is triangular without or with a vertical keel at midsection depending on whether the cross-sectional elongation ratio μ is smaller or larger than the critical value $\mu = 0.206$. The lift-to-drag ratio of the optimum body increases as the elongation ratio of the cross section decreases. For a Newtonian pressure distribution and a surface-averaged skin-friction coefficient $C_f = 10^{-3}$, the highest attainable lift-to-drag ratio is $E = 5.29$ and the corresponding thickness ratio is $\tau = 0.126$. A paper based on Ref. 10 has appeared in Astronautica Acta (Ref. 14).

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